International Journal of Theoretical & Applied Sciences, 7(2): 14-20(2015)

ISSN No. (Print): 0975-1718
ISSN No. (Online): 2249-3247

Transfer Matrix Method for Noise Attenuation on Single Expansion Chamber Muffler having Central Inlet and Central Outlet with Experimental Techniques and FEA Validation

Amit Kumar Gupta* and Dr. Ashesh Tiwari**

**Assistant Professor, Mechanical Engineering Department, IET-Devi Ahilya University, Indore, (MP), INDIA*
***Professor, Mechanical Engineering Department, IET-Devi Ahilya University, Indore, (MP), INDIA*

(Corresponding author: Amit Kumar Gupta)
(Received 11 July, 2015 accepted 17 August, 2015)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: Noise reduction on muffler is generally calculated by transmission loss and insertion loss. Single expansion chamber generally used as silencing elements in intake and exhaust systems. The main aim of this paper is to use of mathematical model like Transfer Matrix Method which is based on plane wave propagation. Also an experimental method and FEA tools (Wave 1-D & Comsol Multiphysics) is incorporate to validate the result. Set up of two load method is used here for Transmission Loss Measurement. The best used parameter to evaluate the sound radiation characteristics of muffler is transmission loss (TL). This is the one of the most frequently used criteria of muffler performance because it can be predicted very easily from the known physical parameters of the muffler. By comparing the experimental results with FEA results shows the validation of results. This paper is exploring the various ways to calculate transmission loss.

Keywords: Transfer Matrix Method (TMM), Transmission Loss (TL), Two Load Method, Wave 1-D, Comsol.

I. INTRODUCTION

Single expansion chamber abbreviated as reactive mufflers are used to reduce noise level related with internal combustion engine exhausts, compressors and fans. With increasingly stringent regulations for controlling noise pollution of automotive vehicles here mufflers are important part of engine system and commonly used in exhaust system to minimize noise caused by exhaust gases. Designs of muffler have a complex function that affects the noise characteristics and fuel efficiency of the vehicle. Obviously most commonly used parameter to evaluate the sound radiation characteristics of muffler is Transmission Loss. Experimentally two-load method is commonly used to predict the transmission loss of an acoustic muffler. Finite element analysis tools are used to show the comparative study to predict the Transmission Loss of muffler. These numerical methods are allowing the analysis of all types of acoustic mufflers. Finite Element Analysis tools may not be full proof due to many reasons such as modeling, meshing errors etc. It also incorporate with certain assumptions while solving the mathematical partial differential equations like insufficient boundary conditions, insufficient constraints, types of meshing elements, size of meshing. But numerical methods are very useful for optimization of model of having complicated shapes and also where the cost is involved. So that it is essential to optimize the model by finite element analysis and validate it by experimental methods. Validation of experimental setup it is necessary to test the results of model of which analytical, numerical results are known [1]. The measured transmission losses are compared with finite element analysis simulation. It describe that the transmission losses can be determined reliably with the test rig setup. Many tools are available to simulate the transmission loss characteristics of a muffler. In this paper, muffler is simulated by Finite Element Analysis tool like Ricardo Wave – 1D and Comsol is used to predict muffler's transmission loss performances as well transmission loss also predicted by two load method. Hence it is concluded that the experimental measurements and Finite Element Analysis results can employed to any shape and size of muffler [2].

II. MODELING OF SINGLE EXPANSION CHAMBER MUFFLER

A single expansion chamber consists which consist central inlet and central outlet cylindrical tube. These two tubes are mounted at end of cylindrical expansion chamber. The external tube is 500 mm long and 130 mm diameter and the inlet and outlet tube has a diameter of 35 mm shown in Fig. 1.

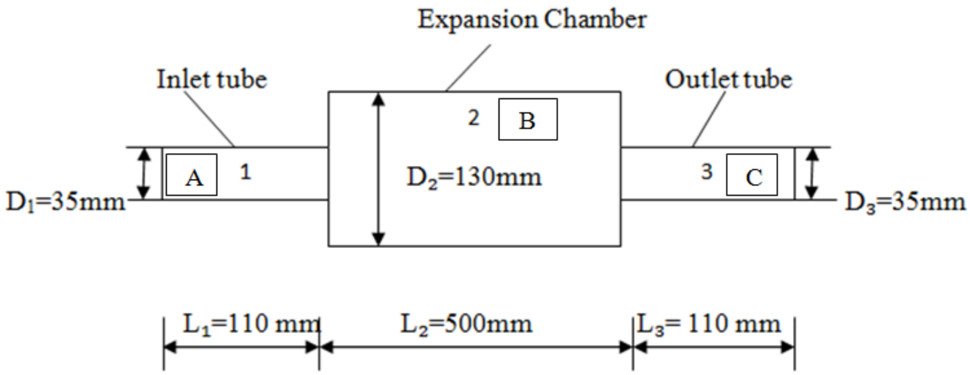

Fig. 1. Geometry of Single Expansion Chamber Test Muffler.

III. MODELING OF TRANSFER MATRIX METHOD (4 POLE PARAMETER)

A simple expansion chamber muffler is the most basic muffler configuration. It is used as a benchmark for more complex muffler configurations. It is, therefore, analyzed here explicitly in order to derive its transmission loss expression, draw its TL curve against the non dimensional frequency,$(k_0 l)$, and discuss its characteristic domes and troughs [3].

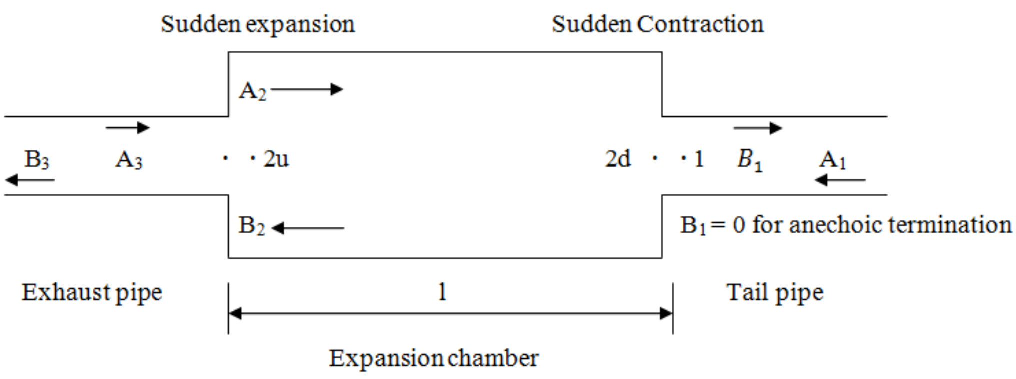

Fig. 2. Schematic of a Simple Expansion Chamber Muffler.

The classical method of analysis proceeds as follows.
Equating pressure and mass velocity at the sudden expansion,

$$A_3 + B_3 = A_2 + B_2 \quad \textbf{(1)}$$

$$\frac{A_3 - B_3}{Y_3} = \frac{A_2 - B_2}{Y_2} \quad \textbf{(2)}$$

Equating pressure and mass velocity at the sudden contraction

$$A_2 e^{-jk_0 l} + B_2 e^{jk_0 l} = A_1 \quad \textbf{(3)}$$

$$\frac{A_2 e^{-jk_0 l} - B_2 e^{jk_0 l}}{Y_2} = \frac{A_1}{Y_1} \quad \textbf{(4)}$$

Assuming the exhaust pipe diameter to be equal to that of the tail pipe,

$$S_3 = S_1 \rightarrow Y_3 = Y_1 \quad \textbf{(5)}$$

Equations 3 and 4 can be solved simultaneously in order to get A_2 and B_2 in terms of A_1, and then Equations 1 and 2 may be solved in order to obtain A_3 in terms of A_1.

Finally, $\text{TL} = 20\log\left|\frac{A_3}{A_1}\right|$. **(6)**

The transfer matrix method consists in multiplying the transfer matrices of sudden expansion, expansion chamber (a uniform tube) and sudden contraction, successively, in order to obtain the overall transfer matrix [4]. Thus,

$$\begin{bmatrix} p_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos k_0 l & jY\sin k_0 l \\ j/Y_2 \sin k_0 l & \cos k_0 l \end{bmatrix} \left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{bmatrix} p_1 \\ v_1 \end{bmatrix}. \quad \textbf{(7)}$$

$$= \begin{bmatrix} \cos k_0 l & jY_2 \sin k_0 l \\ j/Y_2 \sin k_0 l & \cos k_0 l \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \end{bmatrix}.$$

Making use of TL Equation, we get

$$\text{TL} = 20\log\left|\frac{\cos k_0 l + j\frac{Y_2}{Y_1}\sin k_0 l + j\frac{Y_3}{Y_2}\sin k_0 l + \frac{y_3}{Y_1}\cos k_0 l}{2}\right|. \quad \textbf{(8)}$$

IV. MATHEMATICAL MODELLING ON TRANSMISSION LOSS OF MUFFLER

The attenuation in acoustic intensity as an acoustic pressure plane wave propagates outwards from a source is known as Transmission loss. Fig. 3 shows the layout of muffler. Transmission Loss (TL) of a reactive muffler is not necessarily directly indicate to the noise reduction that will be experienced when the muffler is installed; it is useful to compare the TL performance for various expansion chamber sizes as the same trends will be observable in their noise reduction performance. The simple expansion chamber is convenient model to represent the principles of Transmission Loss analysis.

$$TL = 10\log_{10}\left[\frac{Pi}{Pt}\right]^2 = 10\log_{10}\left[\frac{Ai}{At}\right]^2 dB \text{........}(i)$$

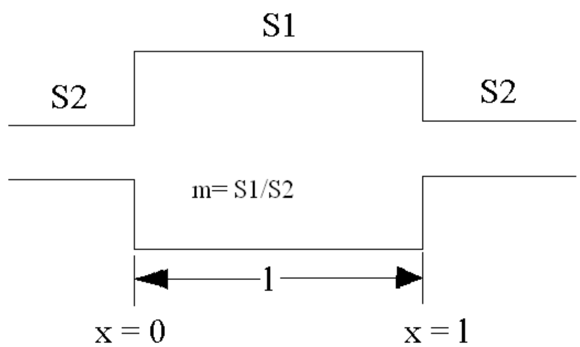

Fig. 3. Schematic layout of Expansion Chamber.

The total sound pressure in the inlet pipe, which is the sum of the incident pressure, p_I and the pressure, p_R reflected from the expansion chamber entrance, may be written using the harmonic pressure solution

$$P_{inlet} = A_I e^{j(\omega t - kx)} + A_R e^{j(\omega t + kx + \beta_1)} \text{.........(ii)}$$

The total sound pressure in the expansion chamber may be written in terms of the right travelling wave and the reflected left travelling wave as

$$P_{exp} = A_A e^{j(\omega t - kx + \beta_2)} + A_B e^{j(\omega t + kx + \beta_3)} \text{...........(iii)}$$

The total sound pressure in the exit pipe may be written as

$$p_T = A_T e^{j(\omega t - kx + \beta_4)} \text{.............(iv)}$$

Continuity of acoustic pressure and volume velocity at the junction of the inlet pipe and the expansion chamber where the coordinate system origin, $x = 0$ will be gives

$$A_I + A_R e^{j\beta_1} = A_A e^{j\beta_2} + A_B e^{j\beta_3} \text{.........(v)}$$

$$S_1 (A_I - A_R e^{j\beta_1}) = S_2 (A_A e^{j\beta_2} - A_B e^{j\beta_3}) \text{........(vi)}$$

At the junction of the expansion chamber and exit pipe, at x = L, continuity of acoustic pressure and volume velocity can given by

$$A_T e^{-jkL + j\beta_4} = A_A e^{-jkL + j\beta_2} + A_B e^{jkL + j\beta_3} \text{...........(vii)}$$

$$S_I A_T e^{-jkL + j\beta_4} = S_2 (A_A e^{-jkL + j\beta_2} - A_B e^{jkL + j\beta_3}) \text{.........(viii)}$$

By using Equations (v) to (viii), the transmitted sound pressure amplitude can be written in terms of the incident sound pressure amplitude as

$$A_T = \frac{2A_I e^{JkL} e^{-J\beta_4}}{2\cos kl + j\left[\frac{S_1}{S_2} + \frac{S_2}{S_1}\right]\text{Sin}kl} \text{.................}(ix)$$

Substituting Equation (ix) into (i) gives

$$TL = 10\log_{10}\left[1 + \frac{1}{4}\left(m - \frac{1}{m}\right)^2 \sin^2 kl\right] \text{..............}(x)$$

Equation (x) derived using 1-D wave analysis rather than lumped analysis, so it takes into account the effect of axial modes but it is not valid if cross modes exist in the chamber [3].

V. COMPARISON OF MATHEMATICAL MODELING AND TRANSFER MATRIX METHOD

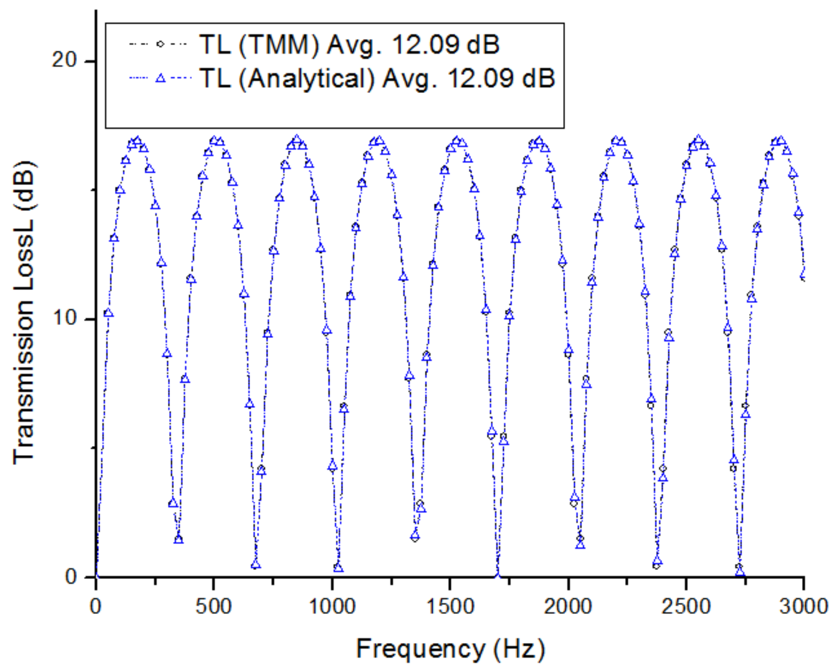

Fig. 4. Schematic layout of Expansion Chamber.

VI. EXPERIMENTAL SETUP

Sound analyzer consists of two assemblies one for input signal (Green Color) which refers to upstream and another for output signal (Red Color) which refers to downstream with computer interfacing. The differences of FFT of these two signals are analyzed in Matlab based sound spectrum software which is developed by autors Amit Kumar Gupta and Dr. Ashesh Tiwari. The difference of upstream and downstream sound pressure level is calculated as transmission loss. The experiment is performed for frequency range of 1 to 3000 Hz.

The readings are taken in two slots with two locations 1-1' and 4-4' which is shown in figure respectively to achieve desired frequency range. The locations 1-2-3-4 are used for measuring pressure in frequency range 10-400 Hz, while the locations 1'-2-3-4' are used for measuring pressure in frequency range of 400-3000 Hz. Two microphones are used for measurement, which are sufficient for measurement of transfer function between sound pressures measured at two locations [2]. All other locations except locations where microphone are inserted are sealed with rubber cap to avoid sound leakage.

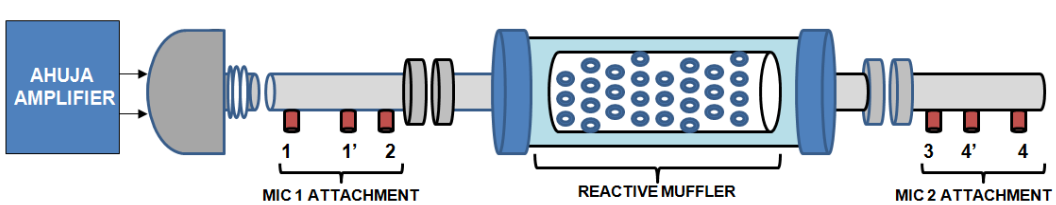

Fig. 5. Schematic Layout of Test Rig.

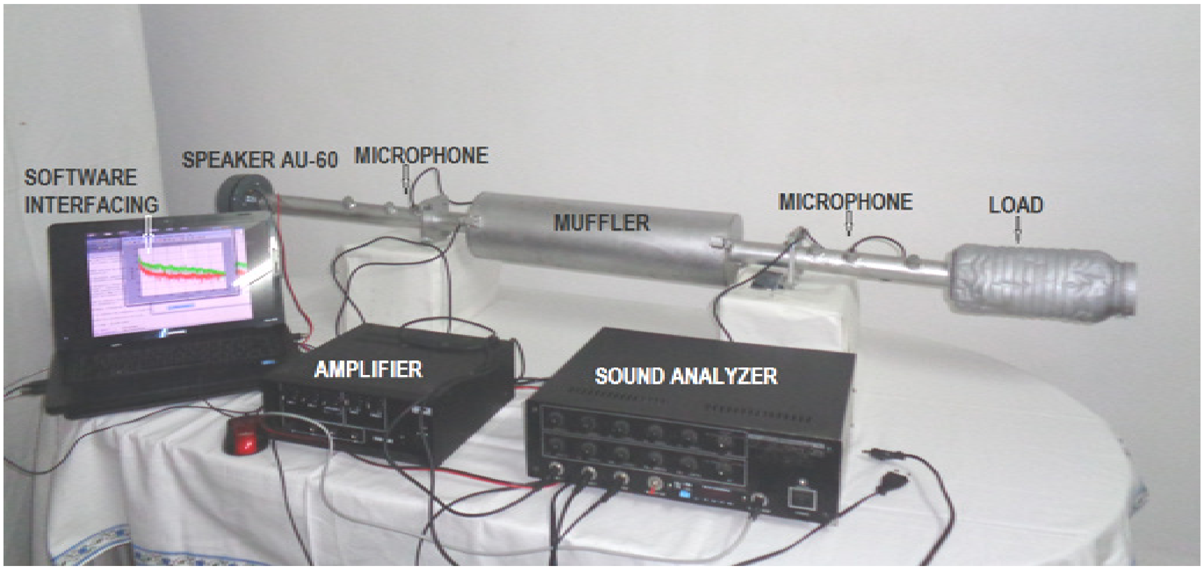

Fig. 6. Experimental Test Set Up.

VII. FEA TOOLS TO CALCULATE TRANSMISSION LOSS

A. Wave 1-D One-Dimensional Wave Approach

WAVE is a 1-dimensional gas dynamics code which is based on finite volume method for simulating engine cycle performance. Tools using this one-dimensional approach accurately predict all engine breathing characteristics. This enables engineers to Consider air system and combustion effects during analysis. One-dimensional tools such as WAVE are also predictive in nature in contrast to the representative nature of mean-value engine models. Since WAVE is an engine specific package, it is very likely to be applied in a co-simulation application where correct representation of the engine dynamics can affect the overall representation of the controller or vehicle dynamics.

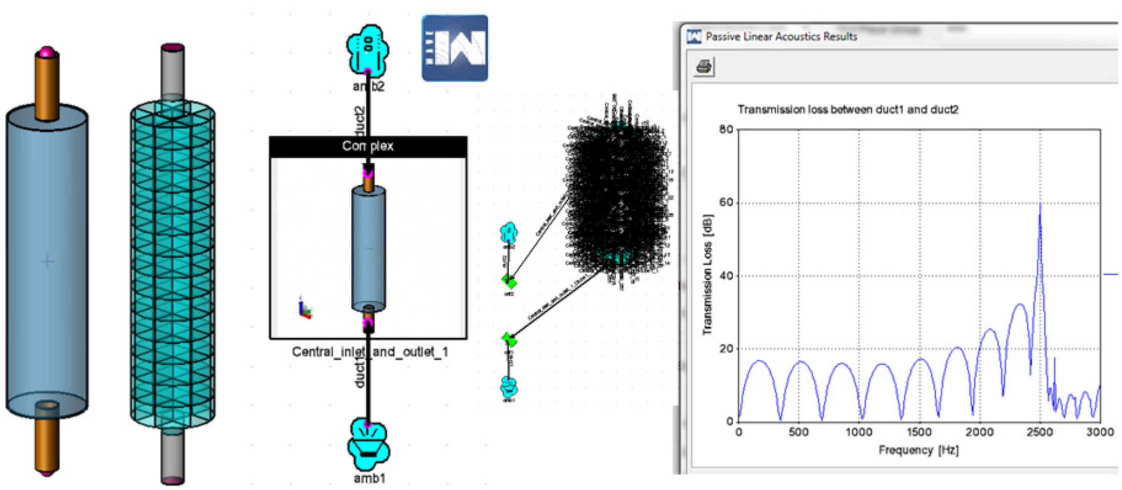

Fig. 7. GUI for Post Processing of Wave 1-D.

The working fluid was perfect air having following boundary conditions:

1. Gas Volume approximately: 6636500 mm^3.
2. Exhaust gas Temperature: 300 K.
3. Exhaust Gas pressure: 1.0 bar.
4. Initial fluid composition: Fresh Air.
5. Upper frequency Limit: 3000 Hz.
6. Lower Frequency Limit: 25 Hz.

Model is prepared on wave build 3D with inlet & outlet boundary condition.

B. COMSOL One-Dimensional Wave Approach

Transmission loss is calculated in COMSOL Multiphysics FEA Tool using the acoustic power at the inlet and outlet of the acoustic system. A model of each muffler design has been developed using COMSOL. Each model was meshed using the default (Lagrange-quadratic) elements with controls applied to achieve a mesh consistent in size and distribution to that generated in the corresponding Wave 1-D model [4]. A harmonic pressure of 1 Pa was specified at the inlet and a radiation condition applied at inlet and outlet. Acoustic damping was not applied at the fluid-structure boundaries for comparison with the Wave 1-D results.

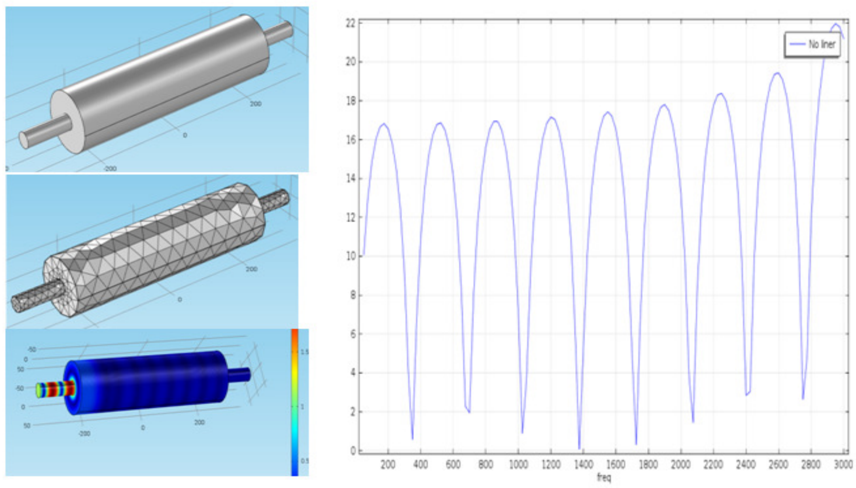

Fig. 8. GUI for Post Processing of Comsol Multiphysics.

VIII. COMPARISON OF ALL THE RESULT

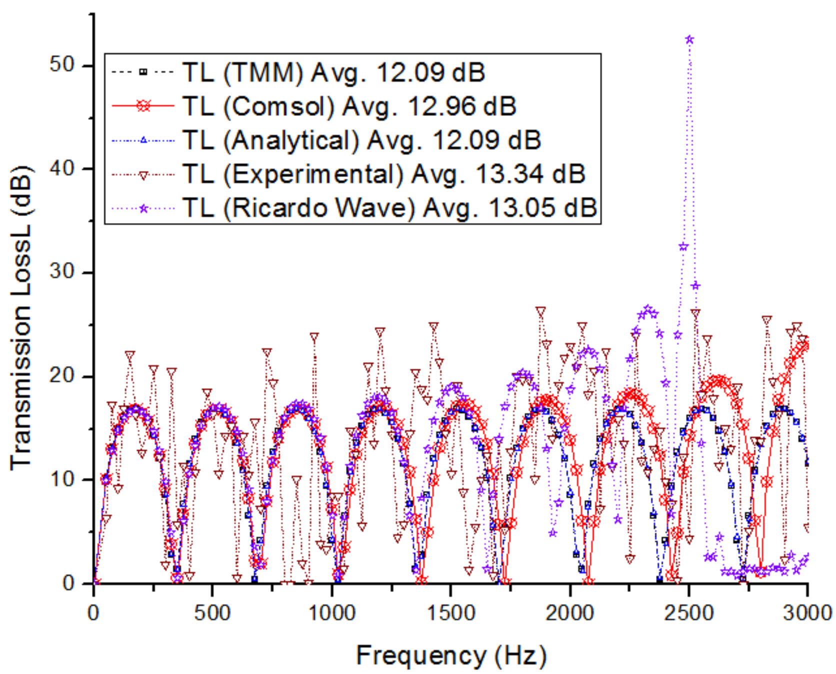

Fig. 9. TL Comparison with Analytical, TMM, Experiment and FEA.

Table 1: Average TL Results upto 3000 Hz.

Average TL (TMM)	Average TL (Analytical)	Average TL (Experimental)	Average TL (Ricardo Wave 1-D)	Average TL (Comsol FEA)
12.09 dB	12.09 dB	13.34 dB	13.05 dB	12.96 dB

IX. CONCLUSIONS

Attenuation curves represent among five observations clearly shows that by the comparison with five results transfer matrix method, analytical, experimental (two load method) and FEA tools like Ricardo wave 1-D & cosmol multiphysics the transmission loss are equally are comparable. Small deviation is appeared with FEA tool is due to meshing parameter. Comparison of additional FEA tools like wave 1-D and comsol results shows the good agreement between existing TMM and analytical method. It also describes the experimental two load method which is used for result comparison. Now any shape of muffler can be modeled to predict the TL measurement. In recent scenario so many complicated geometry where the practical analysis proves too expensive & complicated. Therefore the FEA Tool can be the best approach to achieve the expected outcomes regarding the transmission loss of Muffler.

REFERENCES

[1]. Amit Kumar Gupta, Dr. Ashesh Tiwari "Comparison of Existing Experimental Results with Different Types of Simulation Software for Transmission Loss Estimation of Muffler", March 2015(Pp 17-20), Vol. **2**, Issue 1,*Trends in Machine Design*, March 2015.

[2]. Tao Z. and Seybert A.F. "A review of current techniques for measuring muffler transmission loss" SAE 2003.

[3]. David A. Bies, "Engineering Noise Control Theory and practice Fourth edition" spon press.

[4]. Barbieri R. & Barbieri N. "Finite element acoustic simulation based shape optimization of a muffler" 2006; 346–12p.